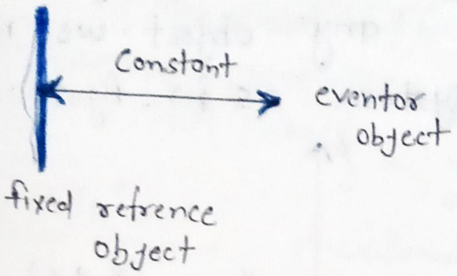
Relative Mechanics

State of Rest :-

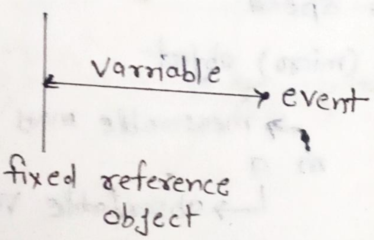


from figure if the distance b/w reference object and event is constant then object or event is in rest.

Acceleration for rest condition, $a = \frac{V_f - V_e}{\Delta t}$

$= \frac{0}{\Delta t} = 0$

State of Motion :-



from figure if the distance b/w reference object and event i.e. varriable them object is in moving condition.

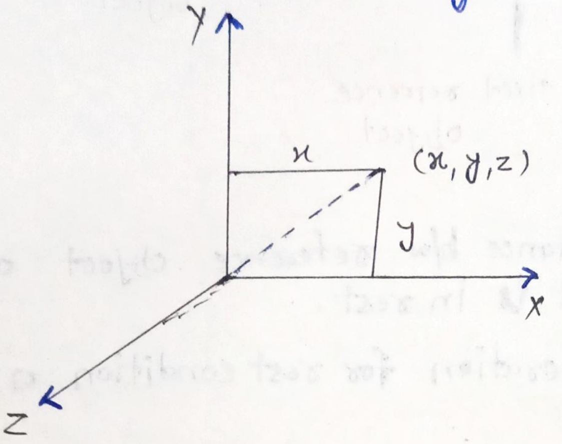
Case. I :→ object moving with the constant speed

$$\boxed{a = 0}$$

Case. II :- Object moving with varriable speed.

$$a = \frac{V_f - V_p}{\Delta t} \neq 0$$

Position of any object → To find the given location or position of any object we must use the co-ordinate system as in figure.



Lack of classical physics →

(i) for slow speed
(ii) for big (micro) object.

$f = m\,a$ → measurable mass ↳ obserbable velocity

* Frame of Refrence → Position co-ordinate (x, y, z) + time → frame of Refrence.

To find the location or position and time of accurance we must use the special set of ~~occurrence~~ co-ordinate system known as frame of refrence.

Types of frame of Refrence :-

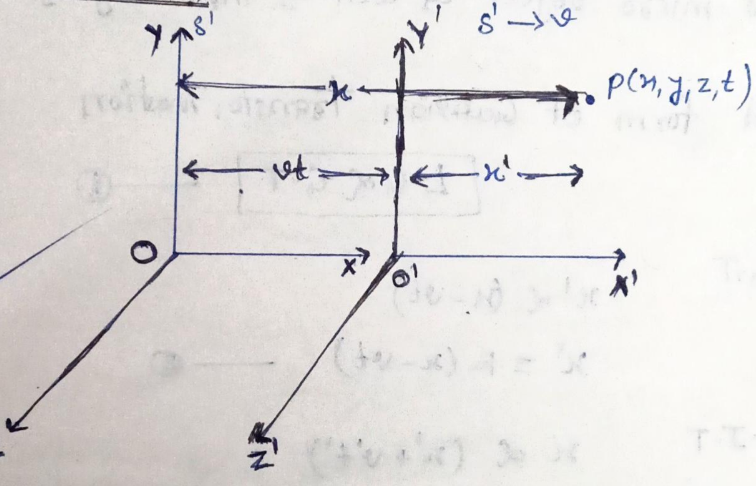
(i) Inertial frame of refrence, $a = 0$
(ii) Non-Inertial frame of refrence, $a \neq 0$

(i) Inertial frame of Refrence -:- unaccelerated ($a=0$) frame is known as Inertial frame of refrence.

ex → building, rest bike, bike moving with the const. speed.

(ii) Non-Inertial frame of Refrence -:- Accelerated ($a \neq 0$) frame is known as ~~kno~~ non-Inertial frame of refrence. eg → earth, revoles around the sun, bike is moving with const velocity.

Galilion Transformation →



Relation b/w two co-ordinate system is known as transformation eqution and for slow speed these eq was derived by Galilian, so known as Galillian Transformation eqation.

from fig :-

$$\left.\begin{aligned} x &= x' + vt \\ y &= y' \\ z &= z' \\ t &= t' \end{aligned}\right\} \quad (1) \qquad G.I.T$$

eq (1) is known as Galilian inverse transformation.

Galilian Ineverse transformation :-

(i) For Inverse Transformation Interchange the co-ordinate and $\vec{v}$ is replaced by $(-\vec{v})$

$$\left.\begin{aligned} x' &= x - vt \\ y' &= y \\ z' &= z \\ t' &= t \end{aligned}\right\} \text{G.T.}$$

Lorentz Transtformation :-

Lorentz transformation equation -

$\left\{\begin{array}{l}\text{fast speed as well as slow speed} \\ \text{for micro object as well as macro object}\end{array}\right\}$

Modified form of Galilian transformation

$$\boxed{L.G \propto G.T} \quad \text{—— (1)}$$

using G.T

$$x' \propto (x - vt)$$

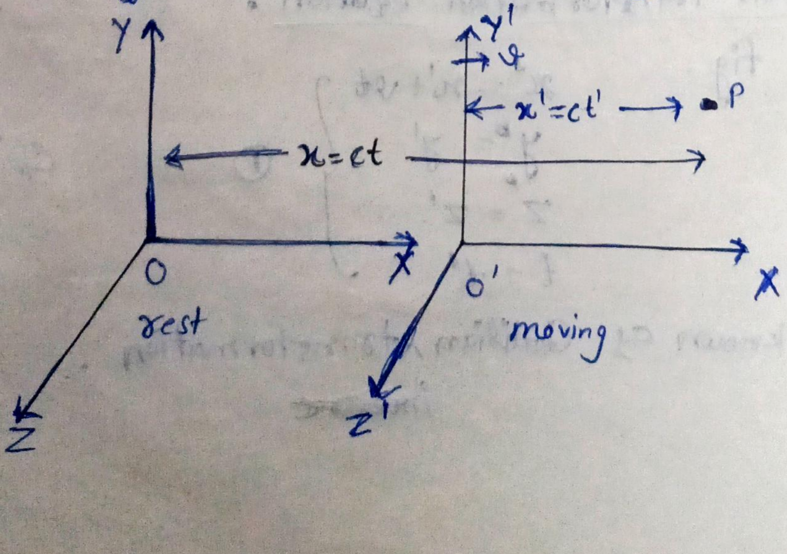
$$x' = k(x - vt) \quad \text{—— (2)}$$

using G.I.T

$$x \propto (x' + v't')$$

$$x = k(x' + vt') \quad \text{—— (3)}$$

from figure $x' = ct'$ $x = ct$ —— (4)



for $O\,(x, y, z, t)$

for $O'\,(x', y', z', t')$

put values of x' and x from eq ④, put eq ② x ③ multiply

$$ct' = k(ct - vt)$$

$$ct = k(ct' + vt')$$

$$c^2 tt' = k^2 tt' (c-v)(c+v)$$

$$\Rightarrow k^2 = \frac{c^2}{(c^2 - v^2)}$$

$$k = \frac{c}{\sqrt{c^2 - v^2}}$$

$$\boxed{k = \frac{1}{\sqrt{1 - v^2/c^2}}}$$

$k \rightarrow$ constant

So eq ② can be written as,

$$\boxed{x' = \frac{1}{\sqrt{1 - v^2/c^2}} (x - vt)} \quad \text{——— (A)}$$

$$\boxed{y' = y} \quad \text{——— (B)}$$

$$\boxed{z' = z} \quad \text{——— (C)}$$

$$t' = x'/c = x' \times 1/c$$

$$t' = \frac{x - vt}{c\sqrt{1 - v^2/c^2}}$$

$$t' = \frac{x/c - vt/c}{\sqrt{1 - v^2/c^2}}$$

$$\boxed{t' = \frac{t - vx/c^2}{\sqrt{1 - v^2/c^2}}} \quad \text{——— (D)}$$

eq (A), (B), (C), (D) known as lorentz transformation

Case I if $v <<<<< c$

$$v^2/c^2 <<<<< 1$$

$$\boxed{x' = x - vt} \quad \text{G.T}$$

Case. II if $v = c$

$$x' = \infty$$

$v = c$ is not possible.

Ether Hypothesis :-

Sound → wave
Sound → Energy
medium → air

Light → wave
Light → energy
medium → eather
- → solid
- → massless
- → Particle less
- → 100% transparrent
- → 100% elastic

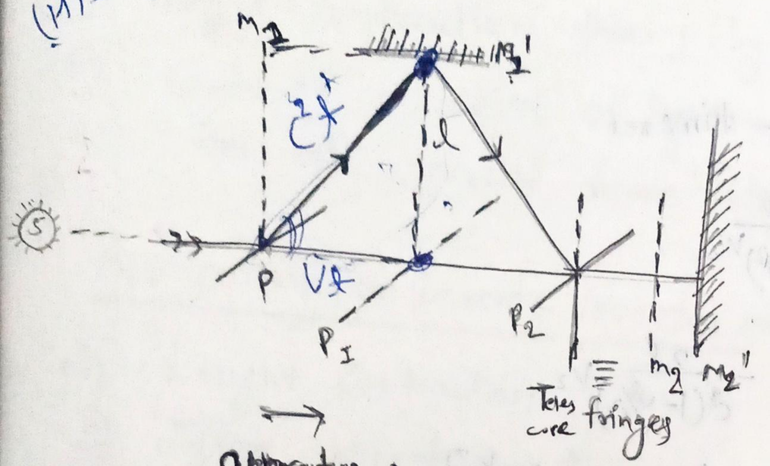
Michelson - Morely Experiment :-

Object → To prove the exitance of eather
or
To find the relative velocity between eather and earth.

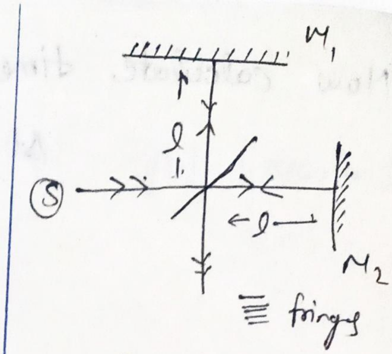
Principal → (i) Interference property of Light.
(ii) Relative velocity b/w two object

Appratus → Michelson Interferometer.

$(H)^2 = (P)^2 + (B)^2$



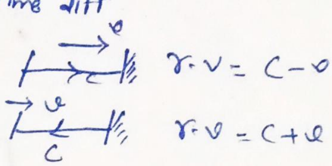
apparatus is moving condition.

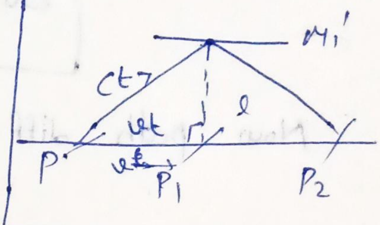


P.d. → 2l − 2l = 0

path diff = nλ (Bright f.)

Time diff





Theory → (Hint : Path difference = time diffrence × c)

from figure;

Time taken by transmitted beam = time in forward direction + time in reverse direction

$$\text{Time}_{trans} = \frac{l}{c-v} + \frac{l}{c+v} = \frac{2lc}{c^2-v^2} \quad \text{—— (1)}$$

from figure using $\Delta PP_1M_1'$ using pythagoras theorem.

$$c^2t^2 = v^2t^2 + l^2$$

$$t^2 = \frac{l^2}{c^2-v^2} \Rightarrow \boxed{t = \frac{l}{(c^2-v^2)^{1/2}}} \quad \text{—— (2)}$$

Total time taken by reflected beam;

$$\text{Time}_{ref} = 2t = \frac{2l}{(c^2-v^2)^{1/2}}$$

Gali

Now calculate time difference,

$$\Delta t = \text{time taken} - \text{time ref}$$

$$= \frac{2lc}{(c^2-v^2)} - \frac{2l}{(c^2-v^2)^{1/2}}$$

$$= \frac{2lc}{c^2(1-v^2/c^2)} - \frac{2l}{c^2(1-v^2/c^2)^{1/2}}$$

Lor

$$= \frac{2l}{c}\left[\left(1-\frac{v^2}{c^2}\right)^{-1} - \left(1-\frac{v^2}{c^2}\right)^{-1/2}\right]$$

$$\boxed{\Delta t = \frac{l v^2}{c^3}}$$

Now path diff -

$$\text{path diff} = \Delta t \times c$$

$$p.d = \frac{l v^2}{c^2}$$

for bright fringes

$$\frac{l v^2}{c^2} = n\lambda$$

$$\boxed{n = \frac{l v^2}{c^2 \lambda}}$$

again this appratus was rotated by 90° path diffrence should be doubled but no change in the observation. we can say that experiment was failed.

Negative results → This experiment could not prove the exitonce of eather.

Negative Result explonation →

(i) Eather drag hypothesis by micheloson morley.

(ii) lenght contraction theory (by lorentz)

(iii) Contancy of speed of light OR velocity of light invarient for all Inertial frame (by einsteen)

Application of Lorentz Transformation:-

(i) Lenght Contraction l

(ii) Time dilaction t

* (iii) Mass variation m

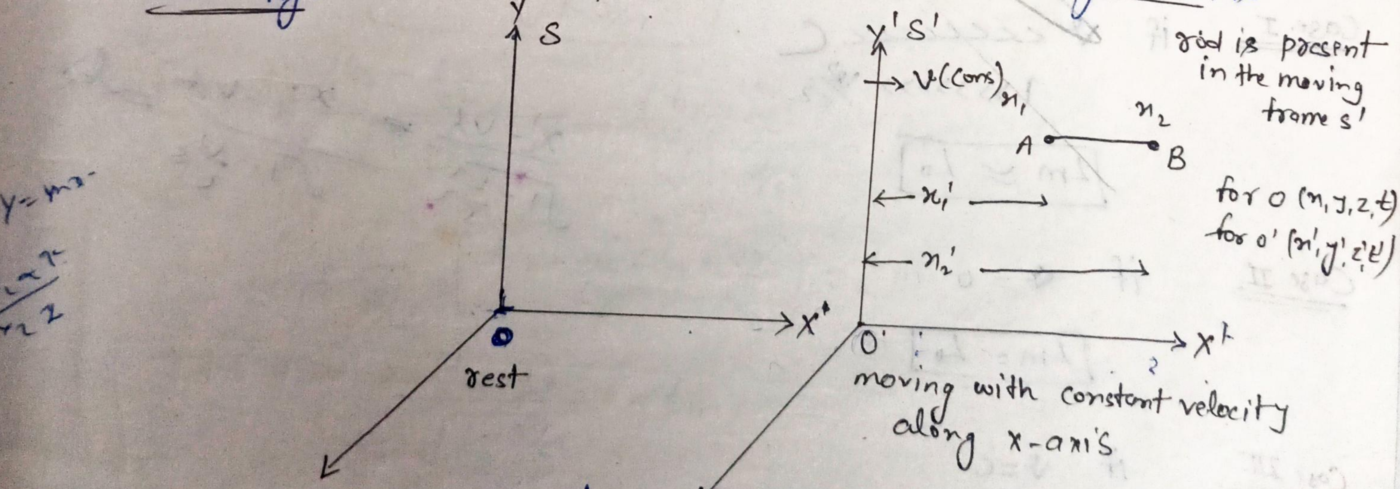
* (iv) velocity addition theorm

* (v) $E = mc^2$

(i) Lenght Contraction -:-

There are two frames S and S' frame S in rest condition and frame S' is moving with const velocity along x-axis.

from fig.



$x_2 - x_1 = lm$

rest lenght $= L_o$

moving lenght $= lm$

(Hint → moving car)

Ga

Lo

Lenght of rod for observer O is

$$(x_2 - x_1) = l_0 \quad \text{--- (1)}$$

lenght of rod for observer O' is

$$x_2' - x_1' = l_m \quad \text{--- (2)}$$

Now apply lorentz transformation, in eq (2)

$$x = \frac{x - vt}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$\frac{x_2 - v.t}{\sqrt{1-v^2/c^2}} - \frac{(x_1 - vt)}{\sqrt{1-v^2/c^2}} = l_0$$

$$\frac{(x_2 - x_1)}{\sqrt{1-v^2/c^2}} = l_0$$

$\because \; l_m = x_2 - x_1$

$$\frac{l_m}{\sqrt{1-v^2/c^2}} = l_0$$

$$\cancel{L_m = L_0\sqrt{1-v^2/c^2}} \quad \boxed{l_m = \frac{l_0}{\sqrt{1-v^2/c^2}}}$$

$l_m \neq l_0$

$l_0 > l_m$

Case. I if $v <<<<<<<<< c$

$1 >>>>>>>> v^2/c^2$

$$\boxed{L_m \simeq L_0}$$

$$\frac{x_2' - vt}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{x_1' - vt}{\sqrt{1-\frac{v^2}{c^2}}} = l_0$$

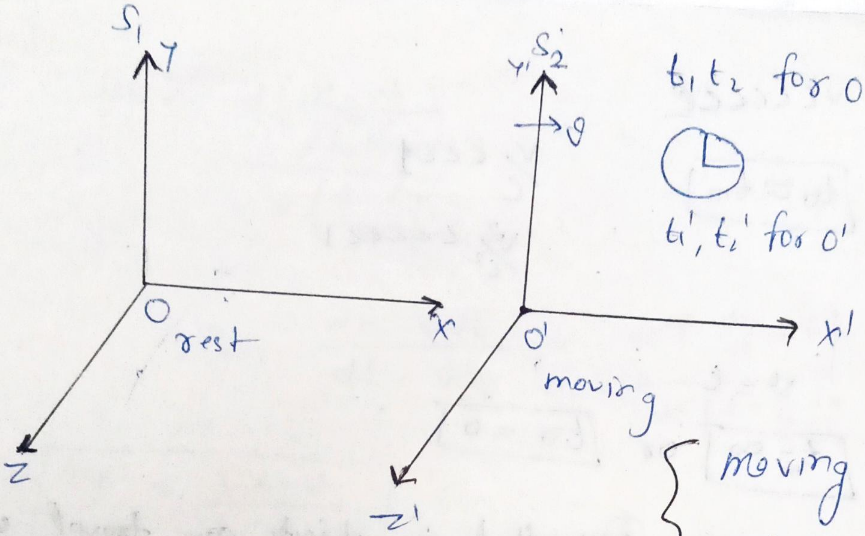
Case II if $v = 0$

$$\boxed{L_m = L_0}$$

Case III if $v = c$

$$\boxed{l_m = 0} \text{ or } \boxed{l_0 = \infty}$$

Time Dilation



moving time $= t_m$
rest time $= t_o$

from figure.

time interval for O is,

$$t_2 - t_1 = t_o \quad \text{——} (A)$$

time interval for O' is

$$t_2' - t_1' = t_m \quad \text{——} (B)$$

Applying lorentz transformation in (B)

$$\boxed{t' = \frac{t - \frac{vx}{c^2}}{\sqrt{1-v^2/c^2}}}$$

$$t - \frac{vx}{c}$$
$$\sqrt{1-\frac{v^2}{c}}$$

$$\frac{\left(t_2 - \frac{vx}{c^2}\right)}{\sqrt{1-v^2/c^2}} - \frac{\left(t_1 - \frac{vx}{c^2}\right)}{\sqrt{1-v^2/c^2}} = t_m$$

$$\frac{1}{\sqrt{1-v^2/c^2}}\left[t_2 - \cancel{\frac{vx}{c^2}} - t_1 + \cancel{\frac{vx}{c^2}}\right] = t_m$$

$$\frac{(t_2 - t_1)}{\sqrt{1-v^2/c^2}} = t_m \qquad \because t_2 - t_1 = t_o$$

$$\frac{t_o}{\sqrt{1-v^2/c^2}} = t_m$$

$$\boxed{t_m = \frac{t_o}{\sqrt{1-v^2/c^2}}} \qquad * \boxed{t_m > t_o}$$

time goes slow

Case I $v <<<<<< c$

$\boxed{t_0 \simeq t_m}$ $v/c <<<< 1$

$v^2/c^2 <<<<< 1$

Case II $v = c$

$\boxed{t_m = \infty}$ or $\boxed{t_0 = 0}$

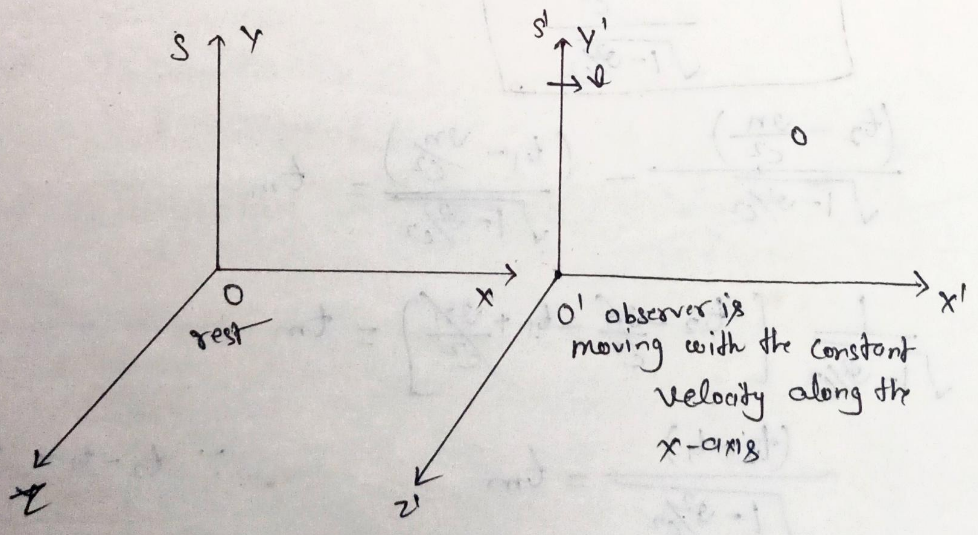
So, we can say that no object can travel with the velocity of light.

Experimental variation of time dilation is $\rightarrow$ ~~train~~ Twin paradox exp.

Case III $v = 0$ $v^2/c^2 <<<<< 1$

$\boxed{t_m = t_0}$

Velocity addition theorem $\rightarrow$ {effect velocity in another velocity}



By the definition of velocity,

$$u_x = \frac{dx}{dt} \text{ --- (1)} \qquad u_x' = \frac{dx'}{dt'} \text{ --- (2)}$$

using L.T in eq ②

$$u_x' = \frac{d\left(\frac{x - vt}{\sqrt{1-v^2/c^2}}\right)}{d\left(\frac{t - vx/c^2}{\sqrt{1-v^2/c^2}}\right)}$$

$$= \frac{dx - v\,dt}{dt - \frac{v\,dx}{c^2}} \Rightarrow \frac{dt\left[\frac{dx}{dt} - v\right]}{dt\left[1 - \frac{v}{c^2}\frac{dx}{dt}\right]}$$

$$\boxed{u_x' = \frac{u_x - v}{1 - \frac{u_x v}{c^2}}} \quad \text{——— (A)}$$

eq (A) is known as velocity addition theorem

velocity inverse transfortation can be written as

$$\boxed{u_x = \frac{u_x' + v}{1 + \frac{v}{c^2}u_x'}}$$

Case I →

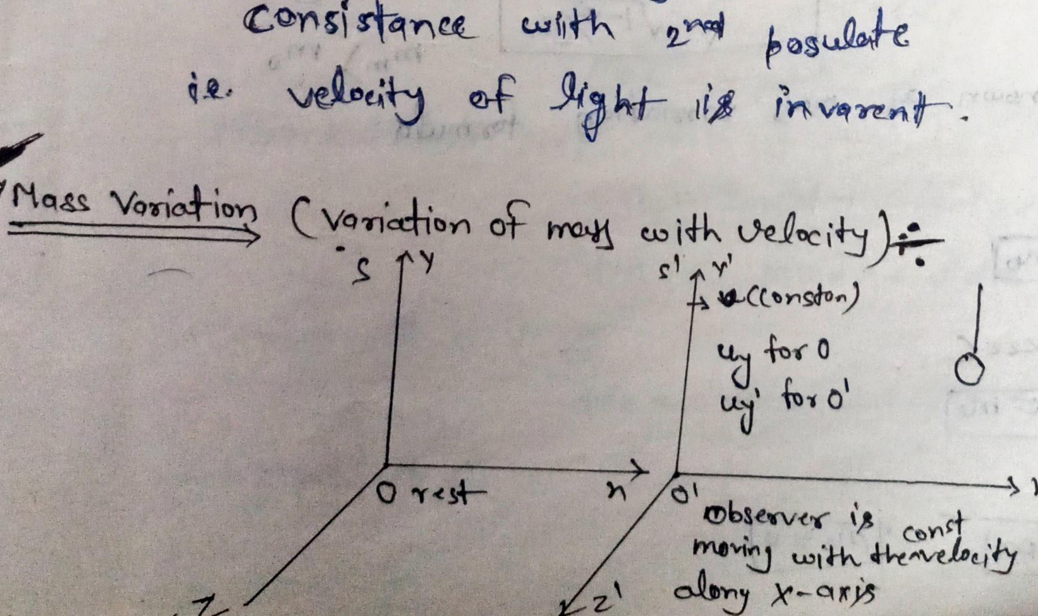
$v = c$, then $u_x = c$

$u_x' = c$, then $u_x = c$

$u_x' = c$ then $v = c$, $u_x = c$

Hence we see that velocity addition theorem is always consistance with 2nd posulate
i.e. velocity of light is invarent.

Mass Variation (variation of mass with velocity) :-



Object moving along Y axis with slow speed.

rest mass = m_0

moving mass = m_m

To find the variation of mass with velocity we must use the conservation law of momentum :

momentum for O,

$$p_1 = m_0 u_y \quad \text{——— (A)}$$

momentum for O',

$$p_2 = m_m u_y' \quad \text{——— (B)}$$

Now apply conservation law of momentum $p_1 = p_2$

$$m_0 u_y = m_m u_y'$$

$$m_0 \frac{dy}{dt} = m_m \frac{dy'}{dt'} \quad \because \text{L.T.}, \ y' = y$$

$$\frac{m_0}{dt} = \frac{m_m}{dt'} \quad \text{——— (C)}$$

Now use time dilation formula $dt' = \frac{dt}{\sqrt{1 - v^2/c^2}}$

So eq (C) written as

$$\frac{m_0}{dt} = \frac{m_m}{dt/\sqrt{1 - v^2/c^2}}$$

$$\boxed{m_m = \frac{M_0}{\sqrt{1 - v^2/c^2}}} \quad \text{——— (D)}$$

$m_m > m_0$

eq (D) is known as mass variation formula

Case I if $v = 0$

$$\boxed{m_m = m_0}$$

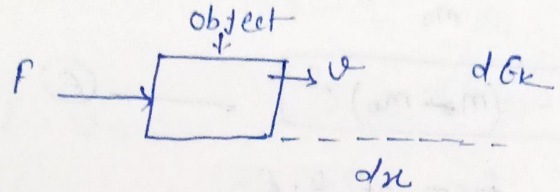
Case II if $v \lll c$

$$\boxed{m_m \simeq m_0}$$

Case III if $v = c$

$$\boxed{m = \infty} \quad \boxed{m_0 = 0}$$

Mass energy relation or mass energy equation ($E = mc^2$)



$v \to$ variable

$m \to$ variable

we know that

work energy theoram can be written as

$$dw = dE_k = f \cdot dx \quad \text{——— (A)}$$

Step 1.

$$\left\{ \because f = ma = \frac{dp}{dt} \right\}$$

$$dE_k = f \cdot dx$$

$$= \frac{dp}{dt} \cdot dx$$

$$dE_k = d(mv) \cdot v \qquad \left\{ \because v = \frac{dx}{dt} \right\}$$

$$dE_k = v\,[m\,dv + v\,dm]$$

$$dE_k = mv\,dv + v^2 dm \quad \text{——— (B)}$$

Step 2

using mass variation formula

$$m_0 = m\sqrt{1 - v^2/c^2}$$

$$(m_0)^2 = m^2 (1 - v^2/c^2)$$

$$\underset{\text{constant}}{m_0^2 c^2} = \underset{\text{vary}}{m^2}\underset{\text{constant}}{c^2} - \underset{\text{variable}}{m^2 v^2}$$

$m \to 1$

$m^2 \to 2m\,dm$

$m^3 \to 3m^2\,dm$

$m^4 \to 4m^3\,dm$

diffrentiate both side

$$0 = 2mc^2 dm - [2m^2 v\,dv + 2v^2 m\,dm]$$

$$c^2 dm = mv\,dv + v^2 dm \quad \text{——— (C)}$$

compare (B) and (C)

$$dE_k = c^2 dm \quad \text{——— (D)}$$

Integreat eq. (D)

$$\int_0^{E_k} dE_k = c^2 \int_{m_0}^{m} dm$$

$$\boxed{E_k = (m - m_0)c^2} \longleftarrow (E)$$

This is relativistic form of K.E.

So we can write $E = E_k + V$

$E_k = E - V \longleftarrow (F)$

$\boxed{E = mc^2}$ & $\boxed{V = m_0c^2}$ rest ~~mass~~ energy.

Experimental verification :-

⇒ Pair production, pair annihilation.

⇒ Nuclear fission & fusio.

Relation Energy & momentum :

$$\boxed{E^2 - p^2c^2 = m_0^2c^4}$$

L.H.S

$E^2 - p^2c^2$

$\Rightarrow (mc^2)^2 - (mv)^2c^2$

$\Rightarrow m^2c^4 - m^2v^2c^2$

$\Rightarrow m^2c^2\,(c^2 - v^2)$

$\Rightarrow m^2\,(1 - v^2/c^2)\cdot c^4$

$\Rightarrow m_0^2c^4$

$$\boxed{E^2 - p^2c^2 = m_0^2c^4}$$

mass less particles :- we know that

$$m = \frac{m_0}{\sqrt{1-v^2/c^2}}$$

if $v = c$

$$\boxed{m_0 = 0}$$

particles which have zero rest mass known as massless particle.

eg → photon, boson particle etc.

* The End *

Numericls

Q-1 what will the lenght of a meter rod appeard to be form a person traveling parllel to the lenght of the road at the spead of 0.8c.

$v = 0.8c$ $\quad L_0 = 1m$ $\quad l_m = ?$

$$l = l_0\sqrt{1-v^2/c^2} \Rightarrow 1\sqrt{1-(0.8)^2c^2/c^2} \Rightarrow 1\sqrt{1-0.64} = \sqrt{0.36}$$

$$\boxed{l_m = 0.6\,m}$$

Q.2 How fast would a rocket have to go relative to an observer to 99% of its lenght at rest.

$L_0 = 100$ m $\quad L_m = 99m$

$l_0 = 100$
$l_m = 99$
$c = 3\times10^8$ m/s

$$L_m = L_0\sqrt{1-v^2/c^2}$$

$$\frac{L_m}{L_0} = \frac{99}{100} = \sqrt{1-v^2/c^2} \Rightarrow v^2/c^2 = 1-\left(\frac{99}{100}\right)^2 \Rightarrow \left(1+\frac{99}{100}\right)\left(1-\frac{99}{100}\right)c^2$$

$$v = \sqrt{\frac{199\times1}{100\times100}c^2} \Rightarrow 0.141\,c \text{ Ans}$$

Imp
Q.3) at what speed should a clock be moved so that it may appear to lose 1 min each houl.

$$\boxed{t = \frac{t_0}{\sqrt{1-v^2/c^2}}} \quad \boxed{t_m > t_0}$$

$t_0 = 1$ hour $= 60$ min

$t_m = \cancel{59}$ min $= 61$ min

$$61 = \frac{60}{\sqrt{1-v^2/c^2}} \Rightarrow 1-\frac{v^2}{c^2} = \left(\frac{60}{61}\right)^2$$

$$v^2 = \left\{1-\left(\frac{60}{61}\right)^2\right\} c^2 \Rightarrow v = \sqrt{\left(\frac{61-60}{61}\right)\left(\frac{61+61}{61}\right) c^2}$$

$$v = \sqrt{\frac{121}{(61)^2} c^2} = \frac{11}{61} c$$

$v = 0.18c$

$v = 0.18 \times 3 \times 10^8$

$\boxed{v = 5.4 \times 10^8 \text{ m/s}}$ Ans

m. Imp
Q.4 [2/8f] A particle of rest mass m_0 moves with velocity $c/\sqrt{2}$ calculate its mass, moventum, kinetic energy & total energy.

rest mass $= m_0$ $\quad v = c/\sqrt{2}$

$$m = \frac{m_0}{\sqrt{1-v^2/c^2}} \Rightarrow \frac{m_0}{\sqrt{1-\frac{c^2}{2c^2}}} \Rightarrow \frac{m_0}{\frac{1}{\sqrt{2}}} = \sqrt{2}\, m_0$$

$\boxed{m = 1.41 m_0}$

momentum $\quad p = mv = \sqrt{2} m_0 \times \frac{c}{\sqrt{2}}$

$\boxed{p = m_0 c}$

Kinetic energy $E_k = (m - m_0)c^2$

$= (\sqrt{2}m_0 - m_0)c^2$

$= (\sqrt{2} - 1)m_0c^2 = 0.41\, m_0c^2$

$E = mc^2 = \underline{\sqrt{2}m_0c^2}$ Ans

Q.5 A mean life time of meson is 2×10^{-8} sec, calculate the mean life of a meson moving with a velocity $0.8c$.

$t_0 = 2\times10^{-8}$ sec $\quad t_m = ? \quad v = 0.8c$

$$t = \frac{t_0}{\sqrt{1-v^2/c^2}} \Rightarrow \frac{2\times10^{-8}}{\sqrt{1-\frac{0.64\times c^2}{c^2}}} \Rightarrow \frac{2\times10^{-8}}{0.6} \Rightarrow \frac{1}{0.3}\times10^{-8}$$

$$\boxed{t = 3.34\times10^{-8}\text{ sec}}$$

Distance $d_s = t_0 v = 2\times10^{-8}\times0.8\times3\times10^{8}$

$\Rightarrow 4.8\,m$

Distance $d_m = t_m v \Rightarrow 3.34\times10^{-8}\times0.8\times3\times10^{8}$

$= 2.672\times3$

$= \underline{7.916\,m}$

m. Imp

2012

Q.6 (A) The total energy of a ~~meson~~ moving meson is ~~extremly~~ exactly twice its rest mass energy. find the speed of mass.

(B) and what speed will the mass of a body be 2 times its rest mass also calculate the momentum and kinetic energy

$$E = 2 \times \text{rest mass Energy}$$

$$m\cancel{c^2} = 2m_0\cancel{c^2}$$

$$\boxed{m = 2m_0}$$

$$\frac{\cancel{m_0}}{\sqrt{1-v^2/c^2}} = 2\cancel{m_0}$$

$$\sqrt{1-v^2/c^2} = \frac{1}{2} \Rightarrow 1 - \frac{1}{4} = \frac{v^2}{c^2}$$

$$v = \sqrt{\frac{3c^2}{4}} \Rightarrow \frac{\sqrt{3}c}{2} \Rightarrow \frac{\overset{0.866}{\cancel{1.732}} \times 3 \times 10^8}{\cancel{2}}$$

$$\boxed{v = 2.598 \times 10^8 \, m/s}$$

momentum $p = mv$

$$= 2m_0 \times 2.25 \times 10^8$$

$$\boxed{p = (4.50 \times 10^8) m_0}$$

$$E_k = (m - m_0)c^2$$

$$= m_0 c^2$$

$$\boxed{E_k = (9 \times 10^{16}) m_0}$$ Ans

Relativistic Relation b/w ~~mass~~ Energy and momentum.

$$E = mc^2 = \frac{m_0}{\sqrt{1-v^2/c^2}} c^2$$

$\because p = mv$

then $v = p/m$

$$E = \frac{m_0 c^2}{\sqrt{1 - \frac{p^2}{m^2c^2}}} \Rightarrow \frac{m_0 c^2}{\sqrt{1 - \frac{p^2c^2}{E^2}}} \Rightarrow \frac{m_0 c^2}{\frac{\sqrt{E^2 - p^2c^2}}{E}}$$

$$\Rightarrow E = \frac{E\, m_0 c^2}{\sqrt{E^2 - p^2c^2}}$$

$$\sqrt{E^2 - p^2c^2} = m_0 c^2$$

$$E^2 - p^2c^2 = m_0^2 c^4$$

$$\boxed{E = \sqrt{m_0^2 c^4 + p^2c^2}}$$